

Papers written by  
Australian Maths  
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**SEMESTER TWO**

**YEAR 12**

**MATHEMATICS SPECIALIST**

**UNITS 3-4**

**2017**

**Section One  
(Calculator-free)**

Name: \_\_\_\_\_

Teacher: \_\_\_\_\_

**TIME ALLOWED FOR THIS SECTION**

Reading time before commencing work:

5 minutes

Working time for section:

50 minutes

**MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION**

**To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: nil.

**IMPORTANT NOTE TO CANDIDATES**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**To be provided by the supervisor**

Question/answer booklet for Section One.

A formula sheet which may also be used for Section Two.

**Structure of this examination**

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
<b>Section One Calculator—free</b>	<b>7</b>	<b>7</b>	<b>50</b>	<b>52</b>	<b>35</b>
Section Two Calculator—assumed	12	12	100	98	65
Total marks				150	100

**Instructions to candidates**

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answer in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

1. (3 marks)

Given the relationship  $x^2 + xy + y^2 = 3$ , find the gradient when  $x = 1$ . (3)

2. (12 marks)

(a) Evaluate  $\int_{\pi/6}^{\pi/3} (\cos^2(x) - \sin^2(x)) dx$ . (3)

(b) Evaluate  $\int_0^{0.5} (e^x - e^{-x})^2 dx$  (2)

(c) Use partial fractions to evaluate  $\int \frac{2x-1}{x^2-x-6} dx$  (4)

(d) Determine  $\int \frac{(\ln(x))^3}{x} dx$  using the substitution  $u = \ln(x)$  (3)

3. (3 marks)

Express  $\sqrt{1+\sqrt{3}i}$  in cis form.

(3)

4. (14 marks)

(a) Simplify

(i) 
$$\frac{(1+2i)(3-i)^2}{10(2+i)} \quad (4)$$

(ii) 
$$\frac{\operatorname{cis}(40^\circ)(\operatorname{cis}(10^\circ))^2}{\operatorname{cis}(120^\circ)} \quad (3)$$



(b) Solve for  $z$

$$z^3 - 4z^2 + 14z - 20 = 0. \quad (3)$$

- (c) Use De Moivre's theorem to prove the identity

$$\sin(4x) = 8 \sin(x) \cos^3(x) - 4 \sin(x) \cos(x) \quad (4)$$

5. (7 marks)

(a) The position vector of a particle is given by  $\mathbf{r}(t) = (1 + \sin(t))\mathbf{i} + (1 - \cos(t))\mathbf{j}$  for any time  $t$ .

(i) Show that  $(x-1)^2 + (y-1)^2 = 1$ . (2)

(ii) Find the simplest expression for speed. (2)

(iii) Find the position and acceleration of the particle at  $t = \pi$ . (3)

6. (9 marks)

(a) The centre of a sphere is  $C(2,3,-1)$  and a plane  $P$  is tangential to the sphere at  $T(4,2,3)$ .

(i) Find the equation and radius of the sphere. (2)

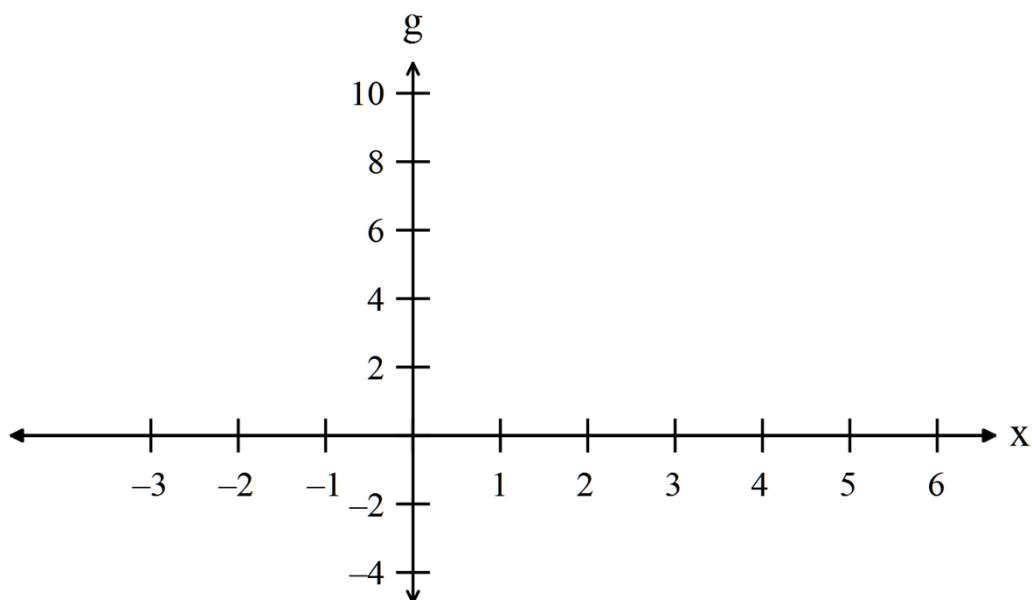
(ii) Find the equation of the plane  $P$ . (3)

(iii) Find two points  $R$  and  $S$  in the plane  $P$  such that  $\mathbf{PR} \times \mathbf{PS} = k \mathbf{CT}$ . (2)

- (b) Find the vector equation of the plane through the points  
 $A(1,1,1)$ ,  $B(2,-1,0)$  and  $C(-1,-2,3)$ . (2)

7. (4 marks)

- (a) Sketch the function  $g(x) = \frac{x(x+2)}{(x-1)}$  on the set of axes below, showing any asymptotes and intercepts. (4)



**END OF SECTION ONE**

***Extra page for working if necessary***

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